6.B/5 $\int_0^1 (2x+1)^4\,dx$

find $\int (2x+1)^4\,dx$

$u = 2x+1$

$\frac{du}{dx} = 2$ so $du = 2dx$

$\frac{1}{2}\int u^4\,du$

$= \frac{1}{2}\left(\frac{u^5}{5}\right)+C$

$= \frac{1}{2}\left(\frac{(2x+1)^5}{5}\right)+C$

② use FTC to evaluate d.I.

$\int_0^1 (2x+1)^4\,dx = \left[\frac{1}{10}(2x+1)^5\right]\Big|_0^1$

$= \frac{1}{10}3^5 - \frac{1}{10}1^5 = \frac{242}{10}$

Let $u = 2x+1$

$du = 2\,dx$ so

$\frac{1}{2}du = dx$

And

$x=0 \Rightarrow u = 2(0)+1 =$

$x=1 \Rightarrow u = 2(1)+1 =$

$\frac{1}{2}\int_1^3 u^4\,du$

$= \frac{1}{10}u^5\Big|_1^3$

$= \frac{1}{10}(3^5 - 1^5) = \frac{242}{10}$

Jan 26-7:28 AM

6.8/9

$$\int_0^8 x\sqrt{1+x}\,dx$$

find $\int x\sqrt{1+x}\,dx$

$$\begin{cases} u = 1+x \\ du = dx \\ u-1 = x \end{cases}$$

$$\int (u-1)\sqrt{u}\,du$$

$$= \int u^{3/2} - u^{1/2}\,du$$

$$= \frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3} + C$$

Let $u = 1+x$, $du = dx$, $x = u-1$

$x=0 \Rightarrow u=1$

$x=8 \Rightarrow u=9$

$$\int_1^9 (u^{3/2} - u^{1/2})\,du$$

$$= \left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_1^9$$

$= \cdots\cdots$

now FTC...

$$\int_0^8 x\sqrt{1+x}\,dx = \left(\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}\right)\Big|_0^8$$

$$= \left(\frac{2}{5}3^5 - \frac{2}{3}3^3\right) - \left(\frac{2}{5}(1) - \frac{2}{3}(1)\right)$$

$$= \frac{486}{5} - 18 - \frac{2}{5} + \frac{2}{3}$$

Jan 26-7:29 AM

7.2... using Definite Integrals to calculate Volumes

30 units^3

$V_{cross\ section} = 2 \cdot 3 \cdot \Delta x$

$= 6\Delta x$

stays 6.

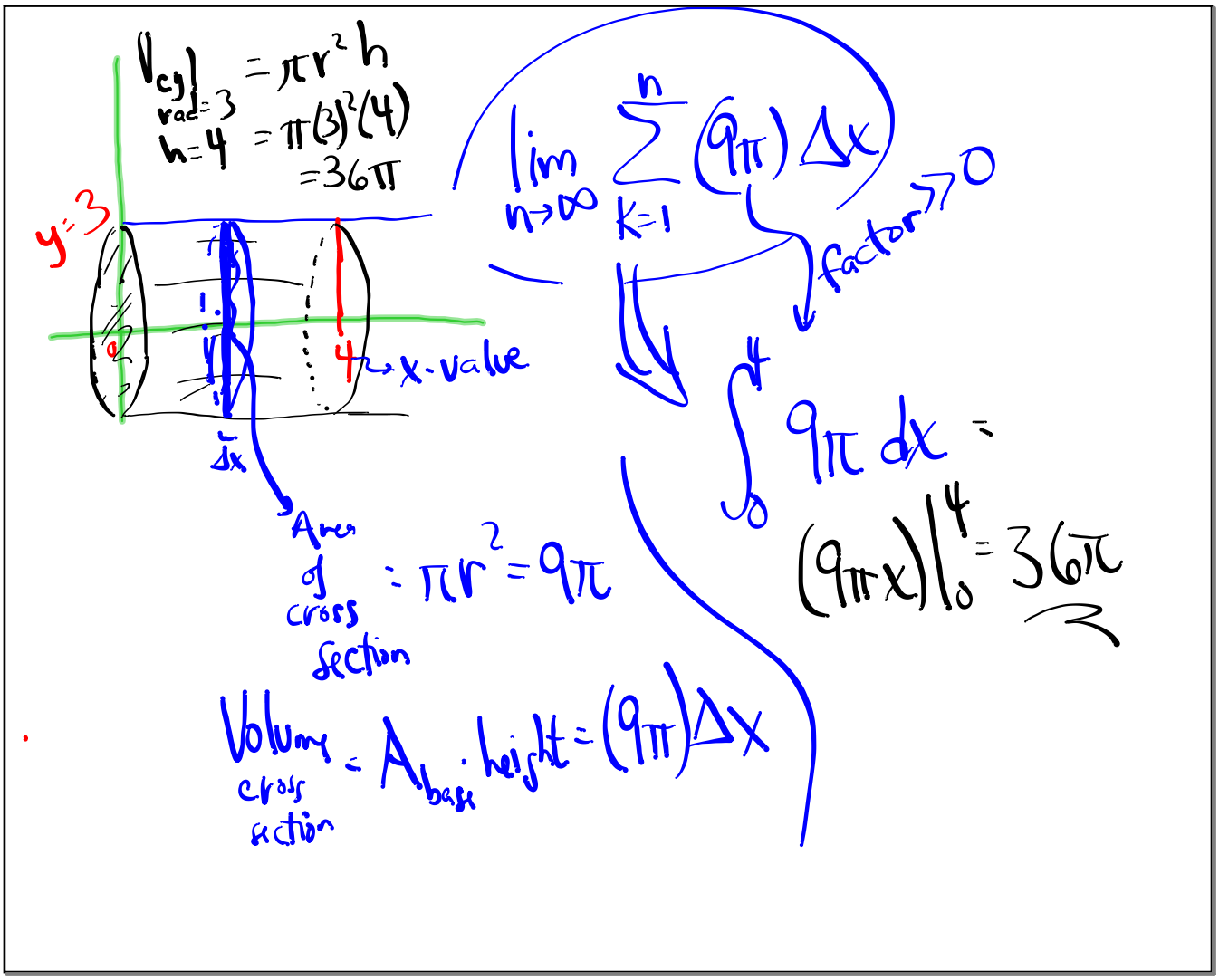
$$\lim_{n\to\infty} \sum_{k=1}^{n} 6 \cdot \Delta x, \quad \Delta x > 0 = \int_0^5 6\,dx$$

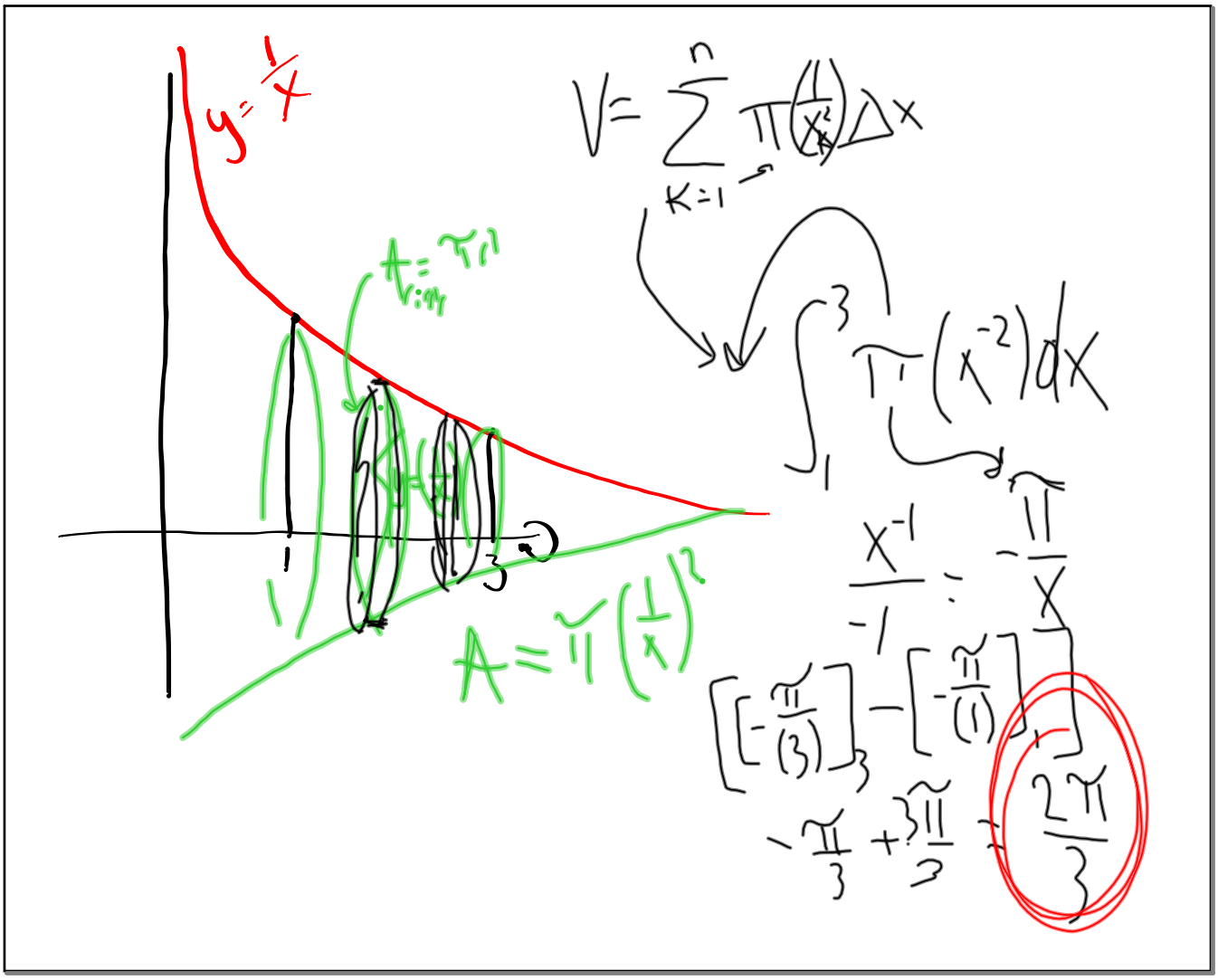
height 6

2x3

$$= (6x)\Big|_0^5 = 30 - 0$$

$$= 30$$

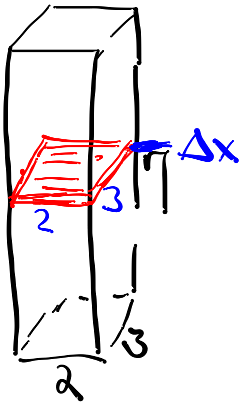

$V_{cyl} = \pi r^2 h$
rad = 3
h = 4
$= \pi(3)^2(4)$
$= 36\pi$
y = 3
4
x-value
Δx
Area of cross section $= \pi r^2 = 9\pi$
Volume cross section $= A_{base} \cdot height = (9\pi)\Delta x$
$\lim_{n\to\infty} \sum_{k=1}^{n} (9\pi)\Delta x$
factor
$\int_0^4 9\pi\, dx =$
$(9\pi x)\Big|_0^4 = 36\pi$



Jan 26-8:26 AM

7.2 / Finding volumes using definite integrals

simple case

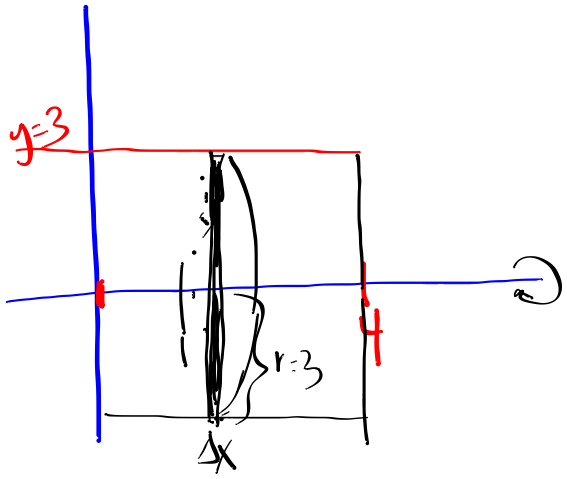


$$V \approx \lim_{n\to\infty} \sum_{K=1}^{n} (2\cdot 3)\cdot \Delta x$$

doesn't go to zero

0 in limit

$$\int_0^7 6\, dx = 6x\Big|_0^7 = 6\cdot 7 - 6\cdot 0$$
$$= 42$$



$V \approx \sum_{k=1}^{n} (9\pi)\Delta x$ (area cross section)

$\int_0^4 (9\pi)\,dx$

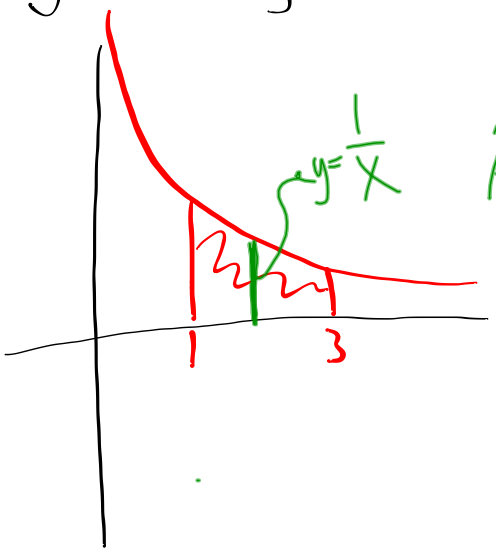
$= 9\pi x \Big|_0^4 = 36\pi$

$V_{\text{cross section}} = \pi r^2 h$

$= \pi(3)^2 \Delta x = 9\pi\Delta x$

Jan 26-8:26 AM

Find the volume of the solid formed by rotating $y = \frac{1}{x}$; $x \in [1,3]$ around the x-axis.



$A_{\text{cross section}} = \pi r^2$

$= \pi \left(\frac{1}{x}\right)^2$

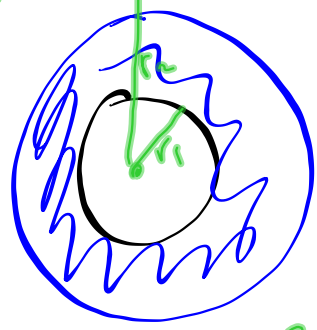
7.2 15

$y = x^2$, $y = 0$, $x = 0$, $x = 2$

Jan 26-12:37 PM

7.2/8 $y=x^2$; $y=x^3$

$\pi\int_0^1 (x^2)^2-(x^3)^2\,dx$



$\pi r_2^2-\pi r_1^2$

r_2: $y=x^2$ r_1: $y=x^3$